Radical Numbers

1. Radical Notation: $a^{1/n} = \sqrt[n]{a}$

Examples:

1)2)

$25^{1/2}$	$8^{1/3}$
$\sqrt{25}$	$\sqrt[3]{8}$
$\sqrt{(5 \cdot 5)}$	$\sqrt[3]{(2 \cdot 2 \cdot 2)}$
5	2

2. Radical Notation: $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Examples:

1)2)

$8^{4/3}$	$9^{3/2}$
$\left(\sqrt[3]{8}\right)^4$	$\left(\sqrt{9}\right)^3$
$\left(\sqrt[3]{(2 \cdot 2 \cdot 2)}\right)^4$	$\left(\sqrt{(3 \cdot 3)}\right)^3$
$(2)^4$	$(3)^3$
$2 \cdot 2 \cdot 2 \cdot 2$	$3 \cdot 3 \cdot 3$
16	27

3. Evaluating: $\sqrt[n]{a^n}$

a) If n is **odd**, then $\sqrt[n]{a^n} = a$

Example:

$\sqrt[3]{-5^3} = -5$

b) If n is **even**, then $\sqrt[n]{a^n} = |a|$

Example:

$\sqrt{-3^2} = |-3| = 3$

c) If a is **positive**, then $\sqrt[n]{a^n} = a$

Examples:

$\sqrt[3]{5^3} = 5$ *and* $\sqrt[4]{5^4} = 5$

4. Rules for Radicals

a) Product Rule: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$

The product of two radicals is the radical of the product.

Examples:

1) $\sqrt{5} \cdot \sqrt{45}$

$\sqrt{5 \cdot 45}$

$\sqrt{5 \cdot 3 \cdot 3 \cdot 5}$

$\sqrt{(3 \cdot 3) \cdot (5 \cdot 5)}$

$3 \cdot 5$

15

2) $\sqrt[3]{4} \cdot \sqrt[3]{8}$

$\sqrt[3]{4 \cdot 8}$

$\sqrt[3]{(2 \cdot 2 \cdot 2) \cdot 2 \cdot 2}$

$2\sqrt[3]{2 \cdot 2}$

$2\sqrt[3]{4}$

b) Quotient Rule: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $(b \neq 0)$

The radical of a quotient is the quotient of the radicals.

Examples:

1) $\sqrt{\frac{5}{36}}$

$\frac{\sqrt{5}}{\sqrt{36}}$

$\frac{\sqrt{5}}{\sqrt{(6 \cdot 6)}}$

$\frac{\sqrt{5}}{6}$

2) $\sqrt[3]{\frac{25}{27x^3}}$

$\frac{\sqrt[3]{25}}{\sqrt[3]{27x^3}}$

$\frac{\sqrt[3]{5 \cdot 5}}{\sqrt[3]{(3 \cdot 3 \cdot 3) \cdot (x \cdot x \cdot x)}}$

$\frac{\sqrt[3]{25}}{3x}$

c) Power Rule: $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

The root of the radical of a radical is the product of their roots.

Examples:

1) $\sqrt[4]{\sqrt{12}}$

$\sqrt[4 \cdot 2]{12}$

$\sqrt[8]{12}$

2) $\sqrt[5]{\sqrt[3]{9}}$

$\sqrt[5 \cdot 3]{9}$

$\sqrt[15]{9}$

5. Rules for Simplifying Radicals

a) No factor under the radical can have a higher power than the root.

$\sqrt[3]{7^5}$

$$\sqrt[3]{7^5} = \sqrt[3]{7^3 \cdot 7^2} = 7\sqrt[3]{7^2}$$

b) No fractions allowed under the radical.

$\sqrt{\frac{2}{25}}$

$$\sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{\sqrt{25}} = \frac{\sqrt{2}}{5}$$

c) No radicals allowed in the <u>denominator</u> (rationalize the denominator).

$\frac{5x}{\sqrt{12}}$

$$\frac{5x}{\sqrt{12}}$$

$$\frac{5x}{\sqrt{4 \cdot 3}}$$

$$\frac{5x}{2\sqrt{3}}$$

$$\frac{5x}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{5x\sqrt{3}}{2\sqrt{3} \cdot 3}$$

$$\frac{5\sqrt{3}\,x}{2 \cdot 3}$$

$$\frac{5\sqrt{3}\,x}{6}$$

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