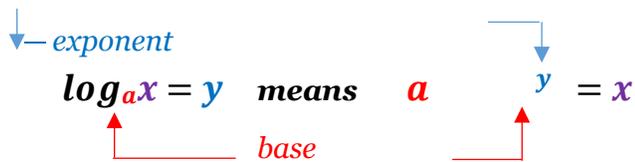


Logarithms

I. A **logarithm** is another way to write an **exponent**.

| Logarithm Form | Exponent Form |
|----------------------|---------------|
| $\log_a x = y$ | $a^y = x$ |
| $\ln_e x = y$ | $e^y = x$ |
| $\log_{10} 1000 = 3$ | $10^3 = 1000$ |
| $\log_2 16 = 4$ | $2^4 = 16$ |
| $\log_3 3 = 1$ | $3^1 = 3$ |
| $\log_5 1 = 0$ | $5^0 = 1$ |



II. **Note:** If the **base** for **log** is left out, it is **understood** to be **10**.
The **base** for **ln** is always **e**.

| Common Logarithm – log <i>Note: $\log x = \log_{10} x$</i> | Natural Logarithm – ln <i>Note: $\ln x = \log_e x$</i> |
|---|---|
|---|---|

III. **Properties of Logarithms:**

| log | | ln |
|--|--------------------------|-----------------------------------|
| $\log_b xy = \log_b x + \log_b y$ | Product Property | $\ln xy = \ln x + \ln y$ |
| $\log_b \frac{x}{y} = \log_b x - \log_b y$ | Quotient Property | $\ln \frac{x}{y} = \ln x - \ln y$ |
| $\log_b x^r = r \log_b x$ | Power Property | $\ln x^r = r \ln x$ |
| $\log_b b^x = x$ $b \log_b x = x$ | Inverses | $\ln e^x = x$ $e \ln x = x$ |

IV. Change-of-Base Theorem:

| | Formula | Example |
|------------|--|---|
| log | $\log_a x = \frac{\log_b x}{a} \log_b$ | $\log_{\pi} 9 = \frac{\log 9}{\log \pi} = 1.9194$ |
| ln | $\log_a x = \frac{\ln x}{\ln a}$ | $\log_{32} 5 = \frac{\ln 5}{\ln 32} = 0.4644$ |

V. Property of Logarithms

| | | |
|---------|-----------------------|-----------------------|
| $x = y$ | <i>if and only if</i> | $\log_a x = \log_a y$ |
|---------|-----------------------|-----------------------|