Exponent Rules

1. Zero-Exponent Rule: $a^0 = 1$

Anything raised **to the zero power** is **1** (exception: 0^0 is undefined).

Examples:

a) $8^0 = 1$

b) $(3x^2y^5z^3)^0 = 1$

2. Power Rule: $(a^m)^n = a^{m \cdot n}$

Same base, keep the base and **multiply** the **exponents**.

Examples:

a) $(xy^2)^4 = x^{1 \cdot 4}\, y^{2 \cdot 4} = x^4y^8$

b) $\left(\frac{x^3}{y}\right)^5 = \frac{x^{3 \cdot 5}}{y^{1 \cdot 5}} = \frac{x^{15}}{y^5}$

3. Negative Exponent Rule: $a^{-n} = \frac{1}{a^n}$ and conversely $\frac{1}{a^{-n}} = a^n$

Move negative exponents to the **other side** of a fraction to make them **positive**.

Examples:

a) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

b) $\frac{x^{-2}\, y^5}{y^5\, z^3} \cdot \frac{z^{-3}}{1} = x^2$

4. Product Rule: $a^m \cdot a^n = a^{m+n}$

Same base, keep the base and **add** the **exponents**.

Examples:

a) $x^2 \cdot x^4 = x^{2+4} = x^6$

b) $2^2 \cdot 2 \cdot 2^3 = 2^{2+1+3} = 2^6 = 64$

5. Quotient Rule: $\frac{a^m}{a^n} = a^{m-n}$

Same base, keep the base and **subtract** the **exponents**

Examples:

a) $\frac{a^8}{a^3} = a^{8-3} = a^5 = a$

b) $\frac{x^5 y^2}{x^4 y^6} = x^{5-4} \cdot \frac{y^2}{y^6} = x^{1} y^{2-6} = x y^{-4} = \frac{x}{y^4}$

6. Fractional Exponents: $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Examples:

a) $x^{\frac{5}{4}} = \sqrt[4]{x^5} = \left(\sqrt[4]{x}\right)^5$

b) $27^{\frac{4}{3}} = \sqrt[3]{27^4} = \left(\sqrt[3]{27}\right)^4 = (3)^4 = 81$

Examples of simplifying using the exponent rules.

1) $\frac{(2a^3b^{-2})^{-4}}{(3a^4b^3)^{-3}}$ **Problem** 2) $\frac{\left(-2a^4b^{-1}\right)^4 \left(5a^4b^{-5}\right)^0}{(3a^3b)^2}$

Rule #1

$\frac{\left(-2a^4b^{-1}\right)^4 \cdot 1}{(3a^3b)^2}$

$2^{-4}a^{-12}b^{8}$

$\frac{(-2)^4 a^{16} b^{-4}}{3^2 a^6 b^2}$

3 $\frac{}{}$ $3^{-3}a^{-12}b^{-9}$ **Rule #2**

$\frac{(-2)^4 a^{16}}{3^2 a^6 b^6}$

$b^{8}3^{3}a^{12}b^{9}$

$\frac{(-2)^4 a^{16}}{b^{4}3^{2}a^{6}b^{2}}$

$\frac{}{}$ $4a^{12}$ **Rule #3**

2

$3^{3}a^{12}b^{17}$

$\frac{2^4 a^{12}}{3^3 a^0 b^{17}}$

Rule #4

$\frac{2^4}{27b^{17}}$

Rule #5

$\frac{(-2)^4 a^{10}}{3^2 b^6}$

Simplify

$\frac{16a^{10}}{9b^6}$

$\frac{16}{}$